

Exercise 107

Express the limit as a derivative and evaluate.

$$\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h}$$

Solution

Recall the definition of a derivative.

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(x)}{h}$$

The function in question is

$$f(x) = \sqrt[4]{x}.$$

Take the derivative by using the power rule.

$$f'(x) = \frac{1}{4}x^{-3/4}$$

Plug in $x = 16$.

$$f'(16) = \frac{1}{4}(16)^{-3/4} = \frac{1}{4(2^3)} = \frac{1}{32}$$

Therefore,

$$\lim_{h \rightarrow 0} \frac{\sqrt[4]{16+h} - 2}{h} = \frac{1}{32}.$$